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THE EXPRESSION OF THE n th POWER OF A NUMBER IN TERMS OF THE n th POWERS OF OTHER NUMBERS, n BE- ING ANY INTEGER; AND THE DEDUCTION OF SOME INTERESTING PROPERTIES OF PRIME NUMBERS.

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[Concluded from August-September Number.]

Making $m=7$, we have

$$8^n = 6^{n+1} - \frac{7}{2} \cdot 4^{n+1} + 7 \cdot 2^{n+1} = 6 \cdot 6^n - 14 \cdot 4^n + 14 \cdot 2^n \dots (37),$$

where $n=5, 3$, or 1 .

When m is even and n is odd, or vice versa, (34) becomes

$$\begin{aligned} (m+1)^n = & (m+1)(m-1)^n - \frac{(m+1)(m)}{2} (m-3)^n \\ & + \frac{(m+1)(m)(m-1)}{6} (m-5)^n - \text{etc., } \dots (38). \end{aligned}$$

COR. V. In (21) m may be made equal to n provided that $n! a_1 a_2 \dots a_n$ be added to the second member.

Suppose these changes to be made and denote the result by....(39).

EXAMPLES.

Making $n=3$, in (39), we have

$$(c+a_1+a_2+a_3)^3=(c+a_1+a_2)^3+(c+a_1+a_3)^3+(c+a_2+a_3)^3 \\ - (c+a_1)^3-(c+a_2)^3-(c+a_3)^3+c^3+1.2.3.a_1a_2a_3....(40).$$

In (39), by making $a_1=a_2=....a_n$, and transposing, we have

$$n! a_1^n=(c+na_1)^n-n[c+(n-1)a_1]^n+\frac{n(n-1)}{2}[c+(n-2)a_1]^n-.... \\ +(-1)^nc^n....(41).$$

Making $a_1=1$, we have

$$n!=(c+n)^n-n(c+n-1)^n+....-(-1)^nn(c+1)^n+(-1)^nc^n....(42).$$

Now making $c=0$, we have

$$n!=n^n-n(n-1)^n+\frac{n(n-1)}{2}(n-2)^n-....-(-1)^nn(1)^n....(43).$$

Thus, for $n=4$, we have

$$1.2.3.4=4^4-4.3^4+6.2^4-4.1^4....(44).$$

For $n=6$ and $c=5$, (42) becomes

$$1.2.3.4.5.6=11^6-6.10^6+15.9^6-20.8^6+15.7^6-6.6^6+5^6....(45).$$

SOME INTERESTING PROPERTIES OF PRIME NUMBERS.

In the following $m+1$ is supposed to be a prime number, and m' , m'' , etc. represent integers.

Since each of the binomial coefficients in (25), $+1$ or -1 , is divisible by $m+1$, (25) may be written

$$(c+ma_1)^n+[c+(m-1)a_1]^n+....(c+a_1)^n+c^n=m'(m+1)....(46).$$

That is, the first member, in which c and a_1 may be any integers and n any integer less than m , is exactly divisible by the prime number $m+1$.

Making $c=0$ and $a_1=1$, (46) becomes

$$(m)^n+(m-1)^n+(m-2)^n+....2^n+1=m''(m+1)....(47).$$

In (46), writing $2m$ for m , making $a_1=1$, $c=-m$, and supposing n an even number, we have

$$(m)^n + (m-1)^n + (m-2)^n + \dots + 2^n + 1^n = m'(2m+1) \dots (48),$$

where $2m+1$ is prime, and where n is any even number less than $2m$.

Thus, 17 will exactly divide

$$8^{2x} + 7^{2x} + 6^{2x} + 5^{2x} + 4^{2x} + 3^{2x} + 2^{2x} + 1,$$

when $x=1, 2, 3, 4, 5, 6$, or 7 .

The converse of either of the preceding properties is not always true; we will now deduce some properties which belong exclusively to prime numbers.

According to Wilson's theorem, m' being an integer,

$$1 + n! = m'(n+1) \dots (49)$$

only when $n+1$ is a prime number.

When $n+1$ is a prime number, (42) may be written

$$n! = (c+n)^n + (c+n-1)^n + (c+n-2)^n + \dots + c^n - m'(n+1) \dots (50).$$

Adding 1 to both sides and we readily obtain

$$(c+n)^n + (c+n-1)^n + (c+n-2)^n + \dots + c^n + 1 = m'(n+1) \dots (51),$$

which is true only when $n+1$ is a prime number.

Making $c=0$, and we have

$$n^n + (n-1)^n + (n-2)^n + \dots + 1 + 1 = m'(n+1) \dots (52).$$

That is, S_n+1 is exactly divisible by $n+1$, when the latter is a prime number, and only when it is prime, where $S_n = 1 + 2^n + 3^n + \dots + n^n$.

In (51), by writing $2n$ for n , and $-n$ for c , and reducing, we have

$$2[n^{2n} + (n-1)^{2n} + \dots + 2^{2n} + 1] + 1 = m'(2n+1) \dots (53),$$

which is also true only when $2n+1$ is a prime number.

Subtract $2n+1$ from both members of (53), and divide by $2n+1^x$, we have

$$\{[n^{2n}-1] + [(n-1)^{2n}-1] + \dots + [2^{2n}-1]\} \div (2n+1) = m''.$$

That is, if each of the quantities in the $[\]$ is exactly divisible by $2n+1$, then $2n+1$ is a prime number. This may be considered a generalization of Fermat's Theorem.